II Encuentro RSME-UMA
Ronda, 12-16 diciembre 2022

MECÁNICA GEOMÉTRICA CONTINUA Y DISCRETA

ORGANIZADA POR: MARÍA BARBERO LIÑÁN, SEBASTIÁN FERRARO, DAVID MARTÍN DE DIEGO

Horario

12/12/2022, 15:30-16: Sebastián Ferraro, Instituto de Matemática (INMABB), Departamento de Matemática, Universidad Nacional del Sur (UNS) – CONICET, Bahía Blanca, Argentina. *Una estrategia de paralelización para problemas discretos variacionales.*

12/12/2022, 16-16:30: Marta Farré Puiggalí, University of Antwerp, Belgium. *Nonholonomic systems and the inverse problem.*

12/12/2022, 16:30-17: María Barbero Liñán, Universidad Politécnica de Madrid, Spain. *Symplectic integration of constrained mechanical systems via discretization maps.*

13/12/2022, 17:30-18: Paula Balseiro, Universidade Federal Flumeninese, Rio de Janeiro, Brazil. *Reducción de mapa momento para sistemas no holónomos.*

13/12/2022, 18-18:30: Juan Carlos Marrero, Universidad de La Laguna, Spain. *Unimodularity and invariant volume forms for Hamiltonian dynamics on Poisson-Lie groups.*

13/12/2022, 18:30-19: Manuel Lainz, Instituto de Ciencias Matemáticas, Spain. *An introduction to contact Hamiltonian systems.*

Resúmenes

Sebastián Ferraro. *Una estrategia de paralelización para problemas discretos variacionales*

Los métodos variacionales discretos tienen un desempeño excelente en las simulaciones numéricas de sistemas mecánicos. Detallaremos un procedimiento iterativo para la solución de ecuaciones variacionales con condiciones de borde, correspondientes a Lagrangianos discretos de orden arbitrario. Este consiste en una estrategia de paralelización que aprovecha las capacidades de las GPUs (tarjetas gráficas) o CPUs multinúcleo. Presentaremos como ejemplo de aplicación el problema de navegación de Zermelo así como un problema de optimización de combustible en un problema controlado de cuatro cuerpos en astrodinámica. Discutiremos condiciones para la convergencia de este método. Esta charla se basa en [1, 2], trabajos conjuntos con David Martín de Diego y Rodrigo Takuro Sato Martín de Almagro.

Referencias:

[1] "Parallel iterative methods for variational integration applied to navigation problems", S. J. Ferraro, David Martín de Diego and Rodrigo T. Sato Martín de Almagro. IFAC PapersOnLine (IFAC Conference Proceedings, 7th IFAC Workshop on Lagrangian and Hamiltonian Methods for Nonlinear Control), vol 54-19, 321–326, 2021.

[2] "A parallel iterative method for variational integration", S. J. Ferraro, David Martín de Diego and Rodrigo T. Sato Martín de Almagro. arXiv:2206.08968, 2022.

Marta Farré Puiggalí. *Nonholonomic systems and the inverse problem*

The inverse problem addresses the question of whether or not a system of second order ordinary differential equations is equivalent to a regular Lagrangian system. If this is the case then we say that the system is variational. EDS (exterior differential systems) techniques have been used to identify variational cases. I will present a notion of variationality for constrained systems, in particular nonholonomic systems, and discuss the possible application of EDS techniques in this constrained case.

María Barbero Liñán. *Symplectic integration of constrained mechanical systems via discretization maps*

A new procedure to construct symplectic methods for constrained mechanical systems will be described in this talk. Discretization maps are defined from a retraction map [1] and they allow to adapt the continuous problem to the discretization scheme rather than viceversa [2]. This approach is very useful to obtain numerical integrators for constrained systems that exactly preserve the constraint. Moreover, the methods can be easily extended to the case of non-linear configuration spaces. This is a joint work with David Martín de Diego.

References:

[1] P.-A. Absil, R. Mahony, and R. Sepulchre. Optimization algorithms on matrix manifolds. Princeton University Press, Princeton, NJ, 2008. https://doi.org/10.1515/

[2] M. Barbero Liñán and D. Martín de Diego. Extended retraction maps: a seed of geometric integrators. Found. Comput. Math. (2022). https://doi.org/10.1007/s10208-022-09571-x.

Paula Balseiro. *Reducción de mapa momento para sistemas no holónomos*

En esta charla voy a presentar un proceso de reducción de sistemas no holónomos que admiten ciertos tipos de simetrías y cantidades conservadas. Estudiaremos una reducción del tipo Marsden-Weinstein que lleva en consideración estas cantidades conservadas y el carácter casi-simpléctico del sistema. La charla finalizará con la presentación de algunos ejemplos clásicos. Este es un trabajo en colaboración con M.E. Garcia, C. Tori y M. Zuccalli (Universidad Nacional de La Plata, Argentina).

Juan Carlos Marrero. *Unimodularity and invariant volume forms for Hamiltonian dynamics on Poisson-Lie groups*

In this talk, I will present several relations between the existence of invariant volume forms for Hamiltonian systems on Poisson-Lie groups and the unimodularity of the Poisson-Lie structure. In particular, we will prove that Hamiltonian vector fields on a Lie group endowed with a unimodular Poisson-Lie structure preserve a multiple of any leftinvariant volume on the group. Conversely, we will also see that if there exists a Hamiltonian function such that the identity element of the Lie group is a nondegenerate singularity and the associated Hamiltonian vector field preserves a volume form, then the Poisson-Lie structure is necessarily

unimodular. Finally, we will illustrate our theory with different interesting examples, both on semisimple and unimodular Poisson-Lie groups.

References:

[1] I. Gutierrez-Sagredo, D. Iglesias Ponte, J. C. Marrero, E. Padrón, Z. Ravanpak: Unimodularity and invariant volume forms for Hamiltonian dynamics on Poisson-Lie groups. Preprint arXiv:2207.05511, 2022.

Manuel Lainz. *An introduction to contact Hamiltonian systems*

Contact Hamiltonian systems are a generalization of the Hamiltonian systems of classical mechanics. The action is added as an extra variable in phase space, and symplectic geometry is changed by contact geometry. In this way, we are able to model a large new class of systems. Indeed, symplectic geometry is unable to deal with dynamics where there is energy dissipation. Nonetheless, this is possible in the contact world. In the recent years, there is a growing interest in the applications of contact Hamiltonian systems, including thermodynamics and mechanical systems with dissipation.

In this talk we will introduce the Hamiltonian and Lagrangian formalism, including the variational formulation through the Herglotz principle. We will explain the basic results regarding symmetries, dynamics and geometric properties of this systems. We will also discuss some of the new applications.
