Álgebras de Hopf y categorías tensoriales

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Horario

Jueves 15 de diciembre, 15:30 - 16:00: Daniel Bulacu, *On double wreath quasi quantum groups.*

Jueves 15 de diciembre, 16:00 - 16:30: Juan Cuadra Díaz, *On the existence of integral Hopf orders in twists of group algebras.*

Jueves 15 de diciembre, 16:30 - 17:00: Elisabetta Masut, *Non-existence of integral Hopf orders for twists of several simple groups of Lie type.*

Viernes 16 de diciembre, 12:30 - 13:00: Abdenacer Makhlouf, *Rota-Baxter bisystems, curved $\mathcal{O}$-operator systems and mixed bialgebras.*

Viernes 16 de diciembre, 13:00 - 13:30: Ramón González Rodríguez, *Quasigroupoids and weak Hopf quasigroups.*

Viernes 16 de diciembre, 13:30 - 14:00: Nicolás Andruskiewitsch, *On the double of the Jordan plane (and other fantastic beasts).*

Resúmenes

Daniel Bulacu. *On double wreath quasi quantum groups*
Let H be a quasi-Hopf algebra over a field k. We present conditions for which the wreath algebra structure and the op-wreath coalgebra structure, defined by a wreath k-algebra R over H which is at the same time an op-wreath coalgebra in the category of H-bimodules, afford on $R \otimes H$ a quasi-Hopf algebra structure. Actually, we will uncover how these structures are determined by the so-called pre-Hopf algebras with 1-cycles within the category of Yetter-Drinfeld modules. Concrete examples of this type will be given. (joint work with Blas Torrecillas)

Juan Cuadra Díaz. *On the existence of integral Hopf orders in twists of group algebras*
In [1] and [2], we found an arithmetic difference between group algebras and semisimple Hopf algebras in connection with Kaplansky's sixth conjecture. Namely: *there are complex semisimple Hopf algebras that do not admit an integral Hopf order*. We provided more instances of this phenomenon in [3]. All the examples studied are twists of group algebras in Movshev's way.

In this talk, we will bring a new perspective on this topic. We will present a group-theoretical condition under which a twist of a group algebra admits an integral Hopf order.

The results we will expound are part of the work [4] in collaboration with Ehud Meir (University of Aberdeen, United Kingdom).

References

[1] J. Cuadra and E. Meir, *On the existence of orders in semisimple Hopf algebras.* Trans. Amer. Math. Soc. 368 (2016), 2547-2562.
[2] ______, *Non-existence of Hopf orders for a twist of the alternating and symmetric groups.* J. London Math. Soc. (2) 100 (2019), 137-158.
[3] G. Carnovale, J. Cuadra, and E. Masut, *Non-existence of integral Hopf orders for twists of several simple groups of Lie type.* Accepted in Publ. Mat. ArXiv:2108.12324.
[4] J. Cuadra and E. Meir, *Existence of integral Hopf orders in twists of group algebras.* ArXiv:2211.00097.

Elisabetta Masut. *Non-existence of integral Hopf orders for twists of several simple groups of Lie type*
The first examples of (finite-dimensional) complex semisimple Hopf algebras which do not admit integral Hopf orders appeared in [2] and [3]. The Hopf algebras considered there turn out to be simple Hopf algebras. The following question was posed in [3]: *Does a simple and semisimple complex Hopf algebra admit an integral Hopf order?*

In this talk, we will give a partial negative answer to this question. We will present several new families of simple and semisimple Hopf algebras which do not admit an integral Hopf order. These Hopf algebras will be constructed as Drinfeld twists of group algebras $\mathbb{C}G$, where G is a finite non-abelian simple group belonging to the following families: $\mathbf{PSL}_2(q), {}^2B_2(q)$ and $\mathbf{SL}_3(q)$. Combining our results with two theorems of Thompson and Barry and Ward on minimal simple groups, we will show that for any finite non-abelian simple group G there is a twist Ω for $\mathbb{C}G$ such that the twisted Hopf algebra $(\mathbb{C}G)_\Omega$ does not admit an integral Hopf order.

This talk is based on the joint work [1] with Giovanna Carnovale and Juan Cuadra.

References

[1] G. Carnovale, J. Cuadra and E. Masut *Non-existence of integral Hopf orders for twists of several simple groups of Lie type.* Accepted in Publ. Mat. ArXiv:2108.12324.
[2] J. Cuadra and E. Meir, *On the existence of orders in semisimple Hopf algebras.* Trans. Amer. Math. Soc. **368** (2016), no. 4, 2547-2562.
[3] J. Cuadra and E. Meir, *Non-existence of Hopf orders for a twist of the alternating and symmetric groups.* J. London Math. Soc. (2) **100** (2019), no. 1, 137-158.

Abdenacer Makhlouf. *Rota-Baxter bisystems, curved $\mathcal{O}$-operator systems and mixed bialgebras*
In this talk, we deal with generalizations of the concept of Rota-Baxter operators. We consider Rota-Baxter coalgebras and discuss a dual version of the Rota-Baxter systems defined by T. Brzeziński, then consider Rota-Baxter bisystems related to bialgebras. We show a relationship to coassociative Yang-Baxter pairs (CYBP) that generalize coassociative Yang-Baxter equation (CYBE). Moreover, we introduce a new type of bialgebras (named mixed bialgebras) which are consisting of an associative algebra and a coassociative coalgebra satisfying a compatible condition determined by two coderivations. We investigate coquasitriangular mixed bialgebras and the particular case of coquasitriangular infinitesimal bialgebras, where we give the double construction. Furthermore, we introduce and study curved $\mathcal{O}$-operator systems which generalize both Brzeziński's Rota-Baxter systems and Bai, Guo and

Ni $\mathcal{O}$-operators.
This is a joint work with Tianshui Ma and Sergei Silvestrov.

References

[1] M. Aguiar, Infinitesimal Hopf algebras. Contemporary Mathematics 267, Amer. Math. Soc. (2000): 1-29.
[2] C. M. Bai, L. Guo, X. Ni, $\mathcal{O}$-operators on associative algebras and associative Yang-Baxter equations. Pacific J. Math. 256(2012): 257–289.
[3] T. Brzeziński, Rota-Baxter systems, dendriform algebras and covariant bialgebras. J. Algebra 460(2016): 1–25.
[4] T. Ma, A. Makhlouf, S. Silvestrov, Rota-Baxter cosystems and coquasitriangular mixed bialgebras. J. Algebra Appl. 20 , no. 4 (2021).
[5] T. Ma, A. Makhlouf, S. Silvestrov, Curved $\mathcal{O}$-operator systems, arXiv:1710.05232.
[6] K. Ebrahimi-Fard, Loday-type algebras and the Rota-Baxter relation. Lett. Math. Phys. 61 (2002): 139–147.

Ramón González Rodríguez. *Quasigroupoids and weak Hopf quasigroups*
In this talk we introduce the notion of exact factorization of a quasigroupoid and the notion of matched pair of quasigroupoids with common base. We prove that if (A, H) is a matched pair of quasigroupoids it is posible to construct a new quasigroupoid $\mathsf{A} \bowtie \mathsf{H}$ called the double cross product of A and H. Also, we show that, if a quasigroupoid B admits an exact factorization, there exists a matched pair of quasigroupoids (A, H) and an isomorphism of quasigroupoids between $\mathsf{A} \bowtie \mathsf{H}$ and B.

On the other hand, we prove that the category of finite quasigroupoids is equivalent to the one of pointed cosemisimple weak Hopf quasigroups over a given field $\mathbb{K}$. As a consequence, if $\mathbb{K}$ is algebraically closed, we obtain that the categories of finite quasigroupoids and cocommutative cosemisimple weak Hopf quasigroups are equivalent. Moreover the restriction of the previous equivalence to the category of quasigroups (loops with the inverse property) provides a categorical equivalence between the categories of quasigroups and of pointed cosemisimple Hopf quasigroups over $\mathbb{K}$ and, as in the weak case, if $\mathbb{K}$ is algebraically closed, the category of quasigroups is equivalent to the one of cocommutative cosemisimple Hopf quasigroups.

Finally, if $\mathbb{K}$ is a field, we show that every matched pair of quasigroupoids (A, H) induce, thanks to the quasigroupoid magma construction, a pair $(\mathbb{K}[\mathsf{A}], \mathbb{K}[\mathsf{H}])$ of weak Hopf quasigroups and a double crossed product weak Hopf quasigroups $\mathbb{K}[\mathsf{A}] \bowtie \mathbb{K}[\mathsf{H}]$ isomorphic to $\mathbb{K}[\mathsf{A} \bowtie \mathsf{H}]$ as weak Hopf quasigroups.

References

[1] Alonso Álvarez, J.N., Fernández Vilaboa, J.M. y González Rodríguez, R.: Weak Hopf quasigroups, Asian Journal of Mathematics 20, No. 4, 665–694 (2016).
[2] Alonso Álvarez, J.N., Fernández Vilaboa, J.M. y González Rodríguez, R.: Quasigroupoids and weak Hopf quasigroups Journal of Algebra 568, 408-436 (2021).
[3] González Rodríguez, R.: Weak Hopf quasigroups and matched pairs of quasigroupoids (preprint) (2022).

Nicolás Andruskiewitsch. *On the double of the Jordan plane (and other fantastic beasts)*
The Jordan plane, a well-known deformation of the polynomial ring in 2 variables, has a structure of braided Hopf algebra and as such, it has various associated Hopf algebras, varying with the characteristic of the base field. I will report on recent work on these Hopf algebras and others arising from the classification of Nichols algebras with finite Gelfand-Kirillov dimension. Partially jointly with Héctor Peña Pollastri and François Dumas.