Congreso RSME-UMA
Ronda, 12-16 Diciembre 2022

Classification and width of (hollow) lattice polytopes

Francisco Santos

Dep. de Matemáticas, Estadística y Computación, Univ. de Cantabria, Spain.
E-mail: francisco.santos@unican.es.

Resumen

Lattice polytopes (that is, polytopes with vertices in a lattice, say $\mathbb{Z}^d$) are important both in algebraic geometry and integer optimization. We are particularly interested in *hollow* ones, that is, those with no interior lattice points. The *flatness theorem* states that the maximum lattice width among all hollow convex bodies in $\mathbb{R}^d$ (in particular, hollow lattice polytopes) is bounded by a constant $\mathrm{Flt}(d)$ depending solely on d. For general convex bodies the best current bound is $Flt(d) \leq O(d^{4/3})$ (modulo a polylog term), but for simplices width is known to be bounded by $O(d \log d)$. In contrast, no construction of convex bodies of width more than linear is known.

We will review several recent results related to the width of hollow lattice polytopes; that is, polytopes with all their vertices in the lattice. Among them:

- We show the first constructions of hollow lattice simplices of width larger than their dimension [1, 2].
- We show how width can help classify lattice polytopes, and their algebraic counter-parts. Eg: In dimension four, we have completely classified empty 4-simplices (equivalently, 4-dimensional terminal quotient singularities) [3], completing a partial classification by Mori, Morrison and Morrison (1988).
- We use results on hollow simplices to answer in the positive a question of Chaucer Birkar (2018) about blowups with only epsilon-canonical singularities [4].

Referencias

[1] Giulia Codenotti and Francisco Santos. Hollow polytopes of large width. *Proc. Am. Math. Soc.* **148**(2) (2020), 835–850.

[2] Joseph Doolittle, Lukas Katthän, Benjamin Nill, Francisco Santos. Empty simplices of large width. Preprint, 22 pages, March 2021. arXiv:2103.14925

[3] Óscar Iglesias-Valiño, Francisco Santos. The complete classification of empty lattice 4-simplices. *Rev. Mat. Iberoam.* **37**:6 (2021), 2399–2432.

[4] G.K. Sankaran and Francisco Santos. Blowups with canonical singularities. Geometry & Topology **25**:4 (2021), 2145–2166.